VELOCITY GAIN FOR SOME SELF-REPELLING PROCESSES

NATHANAEL BERESTYCKI

Abstract. We consider one-dimensional Brownian motion and random walk conditioned (in a suitable sense) to have a local time at every point and at every moment bounded by some fixed constant. It is proved, in the random walk case, that the process is ballistic, with an unknown asymptotic speed. A similar behaviour is established in the Brownian motion case; in addition the exact value of the asymptotic speed is computed and involves the first zero of a Bessel function. It turns out that this velocity is approximately 4.55 as high as required by the conditioning. This sort of velocity gain seems to occur in many similar problems, as is demonstrated on a number of examples where Brownian motion is conditioned on some self-repelling behaviour. For instance we consider the case of Brownian motion conditioned on having local time at 0 growing no faster than $\sqrt{(t)}/(\log t)^c$. In this case we show that the a double phase transition occurs at the values $c = 0$ and $c = 1$. For $c > 1$ the process is transient, and for $c < 0$ the process behaves as an unconditional Brownian motion. For $0 < c < 1$ the process displays an interesting recurrent behaviour but with local time growing much slower than required in the conditioning.

Joint work with Itai Benjamini and Harry Kesten.

University of Cambridge

Date: December 28, 2007.